

Financial Frictions, Occupational Choice and Economic Inequality

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We propose a parsimonious extension of the Lucas (1978) model that distinguishes between entrepreneurs that are self-employed and those that are employers. This distinction is important for understanding the impact of financial frictions on entrepreneurship, aggregate output and income distribution, and for understanding the political economy of financial frictions. We find that the removal of financial frictions decreases the self-employment rate from 24% to 11% (with small effects on the number of employers), increases aggregate output by 48%, and has non-trivial effects on income distribution. We also find that while most households benefit from this reform, the majority of employers lose.

JEL Classification: E20, O10, O40

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We propose a parsimonious extension of the Lucas (1978) model that distinguishes between entrepreneurs that are self-employed (do not hire labor outside the household) from those that are employers. Unlike, standard theories of entrepreneurship based on Lucas (1978) or Hopenhayn and Rogerson (1993), our theory can account for the fact that most firms are small and do not hire employees, and that the median entrepreneurial return is no higher than the median wage income.¹ Moreover, we show that a careful modeling of the occupational structure is important for understanding the impact of

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¹See Hamilton (2000), Hurst and Pugsley (2011), Poschke (2013), Vissing-Jorgensen and Moskowitz (2002) for papers documenting facts on entrepreneurial returns.

financial frictions on entrepreneurship, aggregate output, productivity and the distribution of income, and for understanding the political economy of financial frictions.

Our innovation is to model households that are heterogeneous in two skills – working and managerial skills – and to assume that entrepreneurs decide how much time to devote to the supply of labor and/or managerial inputs. Financial frictions are introduced by modeling an endogenous borrowing constraint that limits the amount of capital that entrepreneurs can use, as in Buera, Kaboski and Shin (2011). Our theory implies that some entrepreneurs will rely only on their own labor and managerial inputs and become self-employed entrepreneurs, while other entrepreneurs will decide to hire labor services in the market and become employers. By modeling heterogeneity in the form of two skills the theory can distinguish between a comparative advantage in entrepreneurship (a high ratio of managerial to working skills) and absolute advantage (a high value of both skills). Heterogeneity in absolute advantage implies that both at the top and bottom of the income distribution there are entrepreneurs and workers.

The model economy is calibrated to Brazilian household data and macro aggregates. Differently to the predominant one-skill model in the literature, our theory can account for key household facts on entrepreneurship in Brazil: First, the heterogeneity in entrepreneurial returns is higher than that among workers and the median income of entrepreneurs does not differ from that of workers, which is important for the theory to be consistent with the outside options faced by potential entrepreneurs. Second, the theory can account for household level data on the occupational structure and occupational transitions in Brazil, which is important for disciplining the way financial frictions distort the selection of entrepreneurs. Moreover, our theory can account for the fact that the variation in rates of entrepreneurship between poor and rich countries is explained by the high rates of self-employment in poor countries, a fact that existing theories of occupational choice cannot account for.

We find that the elimination of financial frictions in Brazil would increase aggregate output by 48% and TFP by 9.0%. The increase in TFP is mostly due to the reallocation of capital across entrepreneurs (intensive margin of entrepreneurs) so that the extensive margin of entrepreneurship is not important for the effect of financial frictions on aggregate TFP in our baseline economy.

We find that the calibration of the fixed cost of operation faced by employers is crucial for the quantitative impact of financial frictions on TFP. When

entrepreneurs switch from self-employment to employers, they increase their managerial time from 0.29 to 1, increasing their TFP by 28% upon an occupational switch. Yet, not all entrepreneurs find it optimal to become employers: entrepreneurs with little net worth or with low managerial ability operate at a low scale and may not be able to cover the fixed cost of operation faced by employers. Hence, the fixed cost of operation represents, implicitly, a technology adoption cost. Our calibration procedure pin downs the magnitude of fixed costs by targeting the fraction of employers in the Brazilian population. The calibrated model economy implies that expenditures in fixed costs are about 4.9% of GDP. To evaluate the sensitivity of our results to the value of fixed costs, we calibrate a model economy assuming that employers face a high fixed costs of operation. In particular, we keep all the targets of the baseline calibration but for the fraction of employers in the data. Instead, we target an expenditure share in fixed costs of 6.5% of GDP. We find that the TFP gains of eliminating financial frictions vary enormously across the two calibrated model economies (from 9.0% in the baseline economy to 27.3% in the economy with high fixed costs). The large difference in TFP gains across experiments is mostly due to the variation in TFP gains along the extensive margin of entrepreneurship. While the TFP gains on the extensive margin are 0.1% in the baseline economy, they are 14.3% in the economy with high fixed costs.

In understanding this result, it is important to note that the elimination of financial frictions *increases* the number of employers in the economy with high fixed costs whereas it *decreases* the number of employers in the baseline economy. In the economy with high fixed costs, the number of employers more than doubles (from 0.9% to 1.9% of the population) when financial frictions are eliminated. As a result, both the number and the average productivity of employers increase, which leads to a large increase of TFP along the extensive margin. On the contrary, in the baseline economy the number of employers decreases with the elimination of financial frictions (from 9.2% to 6.8% of the population). We find that the reduction on the number of employers offsets the TFP gain due to a better selection of employers, leading to a small TFP gain along the extensive margin in the baseline economy.

Interestingly, self-employment reduces the output losses caused by credit market imperfections. In a counterfactual experiment in which self-employment is shut down, we find that the negative impact of financial frictions on aggregate output is higher than in the baseline economy (53% versus 48%). Self-employment allows talented households to divide their time between

working and managing. By producing with a fraction of their own managerial and labor inputs, they can produce at a low scale, saving on labor costs, and making more profits than by operating at a larger scale (e.g. with higher managerial input but hiring outside labor). If the profits of self-employed entrepreneurs are higher than their potential wage income, they can accumulate savings at a higher speed, allowing them to operate at a higher scale in the future and to eventually become employers. Financial frictions make this occupational trajectory more likely because they depress the equilibrium wage rate. Talented entrepreneurs can circumvent low wages by using self-employment as a pathway towards becoming an employer. The occupational transitions in the Brazilian household data support this prediction of the theory: the self-employed are nine times more likely to become employers than paid workers are and more than 70% of the transitions into employer status in two consecutive years come directly from self-employment. Our baseline economy matches these facts closely.

While financial frictions lead to a small increase in income inequality, they are important for the sources and the persistence of this inequality. Since individuals with high managerial talent are more likely to be constrained than low-skilled individuals, financial frictions tend to depress the rates of return on skills faced by skilled employers. On the contrary, financial frictions increase the rate of return on capital owned by skilled employers. A variance analysis shows that while returns to (managerial and working) skills account for 31% of the income variance in the baseline economy, they account for more than 80% of the income variance in the economy with no financial frictions. Financial frictions also imply that the returns to managerial ability are positively correlated with capital income. This explains that income inequality is more persistent in an economy with financial frictions.

We assess the welfare gains and losses of reforming capital markets in our calibrated model economy. We assume that the economy is in a steady state and that suddenly there is a once and for all reform that makes the enforcement of credit contracts perfect. We find that while the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Occupational choices are crucial for understanding the distribution of welfare gains from the reform: Among the households that find themselves worse off, about 93% of them would have been entrepreneurs in the period of the reform had the reform not taken place, and 66% would have been employers. We find that the employers benefiting from the reform tend to be of a higher managerial ability than those who oppose it. The reason is that high ability employers are more

likely to operate at an inefficient scale and to gain more from the elimination of enforcement problems. On the other hand, the financial reform hurts many of the lower-skilled employers. The wage hike after the reform makes it unprofitable for these entrepreneurs to hire outside labor. Our theory thus suggests that employers may have a vested interest in maintaining a status quo with low enforcement.

Our paper relates to a large literature in macroeconomics investigating the consequences of financial frictions on aggregate output, entrepreneurship, and productivity. See, for instance, Erosa (2001), Jeong and Townsend (2007), Amaral and Quintin (2010), Buera and Shin (2011), Buera, Kaboski and Shin (2011), Greenwood, Sanchez and Wang (2010), Midrigan and Xu (2014)). Buera, Kaboski and Shin (2015) provide a recent and comprehensive survey of this literature.² Moll (2014) has emphasized that the persistence of entrepreneurial shocks is important for the ability of entrepreneurs to use self-financing in relaxing the borrowing constraints imposed by financial frictions. This observation has motivated researchers, such as Midrigan and Xu (2014), to use micro-level data on plants to evaluate the impact of financial frictions on the allocation of productive resources across *existing* plants. We use *household-level* data to pin down the shock process and, consistently with the plant-level data used by Midrigan and Xu, we find that there are large and persistent differences in the productivity of entrepreneurs. Moreover, we use the fact that our theory is consistent with household data on entrepreneurship and occupational transitions to discipline the predictions of the theory along the *extensive* margin of entrepreneurship. We show that financial frictions have quite different implications across heterogeneous entrepreneurs and, in particular, across employers and self-employed.

Our theory of entrepreneurship builds on the classical model of occupational choice by Roy (1951). Jovanovic (1994) considers an extension of the Lucas model with two dimensional skill heterogeneity to show that the selection of entrepreneurs vary from the one skill model in that entrepreneurs are not necessarily drawn from the the top of the ability distribution. Our paper extends the analysis in Jovanovic by considering a dynamic model with financial frictions in which entrepreneurs face a time allocation prob-

²See Quadrini (2009) for a survey of papers on entrepreneurship. More generally, following the influential work of Restuccia and Rogerson (2008), Guner, Ventura and Xu (2008), and Klenow and Hsieh (2006) there is a literature studying how the misallocation of productive resources due to different sort of inefficient institutions affect aggregate productivity. See a recent survey by Restuccia and Rogerson (2013).

lem. The latter assumption allows our theory to account for own-account workers (entrepreneurs who do not hire paid labor). Gollin (2008) emphasizes the dominant presence of small businesses in developing economies and incorporates own-account workers in a Lucas model. He assumes that entrepreneurs can choose between two technologies, one of which does not involve paid labor. His theory, however, only assumes one-dimensional skill heterogeneity so that all entrepreneurs are drawn from the top of the ability distribution and make more income than workers. D’Erasmus and Boedo (2012) model the informal economy in order to account for the presence of small businesses in developing countries. Hurst and Pugsley (2011) have emphasized the role of non-pecuniary benefits of entrepreneurship in explaining the low returns face by many entrepreneurs. Poschke (2013) shows that when entrepreneurs search for a business idea and face uncertainty about their initial productivity, the payoff to entrepreneurship is a convex function of their realized productivity. The option value of entrepreneurs encourages risk-neutral individuals to enter entrepreneurship even if they face low ex-ante expected returns relative to work. With the exception of Gollin (2008), none of the papers just discussed account for own-account workers. We believe that extending our theory to model how entrepreneurship interacts with the underground economy, non-pecuniary benefits, and the option value of starting a business may prove an interesting avenue for future research. Finally, Cagetti and De Nardi (2006) study how entrepreneurship affects wealth inequality in the US economy. Our focus is on how the distortion on skill prices caused by financial frictions impacts on income inequality.

I. The Model

We consider a small open economy in steady state. The economy is populated by overlapping generations, each generation consisting of a continuum of households. Households are born at age 20, retire at age 60, and die with certainty at age 75. Each household is endowed with one unit of time at every age. Before the retirement age, households decide how much of their time to allocate to working (t_w) or to managerial (t_m) activities. Households differ in working (z_w) and managerial (z_m) abilities. The logarithm of skills evolves stochastically over the life cycle according to (household i at age t)

$$\begin{aligned} \ln(z_{wit}) &= \beta_w X_t + \alpha_{wi} + u_{wit}, \\ \ln(z_{mit}) &= \beta_m X_t + \alpha_{mi} + u_{mit}, \end{aligned}$$

where z_{wit} (z_{mit}) denote the working (managerial) skills of household i at age t , X_t represents a quartic polynomial of age, α_{wi} and α_{mi} represent household fixed effects on working and managerial productivities, and u_{wit} and u_{mit} are life-cycle shocks received at age t by household i . We assume that the fixed effects are drawn from a bi-variate normal distribution at the first period of life of the household (age 20):

$$\alpha = (\alpha_{wi}, \alpha_{mi}) \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_w^2 & \rho_\alpha \sigma_w \sigma_m \\ \rho_\alpha \sigma_w \sigma_m & \sigma_m^2 \end{bmatrix} \right)$$

where ρ_α is the correlation between the two fixed effects across individuals. The mean fixed effect of the distribution of working skills is normalized to 0.

The life-cycle shocks follow the stochastic process

$$u_{jit} = \rho_j u_{jit-1} + \epsilon_{jit}, \text{ for } j = w, m,$$

with $\epsilon_t = (\epsilon_{wt}, \epsilon_{mt})$ jointly drawn from a bivariate normal distribution with correlation coefficient $\text{corr}(\epsilon_{wt}, \epsilon_{mt}) = \rho_{w,m}$. We further assume that α_{ji} and u_{jit} are mutually orthogonal. The assumptions made imply that, at each age, the distribution of skills is log-normal.

PRODUCTION TECHNOLOGY

Following Lucas (1978), output is produced with a constant returns to scale production technology in managerial, labor, and capital inputs. Entrepreneurs can only use their managerial input because there is no market for managers. The supply of the managerial input is equal to the product of the households' managerial ability (z_m) and the time devoted to managing a business (t_m). The output produced by a household supplying $m = z_m t_m$ units of managerial input and using k units of capital and n efficiency units of labor is:

$$(1) \quad Y(m, k, n) = m^\gamma k^\eta n^\theta, \text{ where } \gamma + \eta + \theta = 1.$$

The time allocation decision of entrepreneurs ($t_m \in [0, 1]$) is modeled to introduced self-employment in the Lucas (1978) framework. When $0 < t_m < 1$ entrepreneurs supply *both* managerial and labor inputs to their own businesses. Specifically, the labor input supplied by entrepreneurs to their business is equal to the product of their working ability (z_w) and the time devoted to non-managerial activities ($1 - t_m$). The total labor input used

by an entrepreneur is the sum of the labor supplied by the entrepreneur $((1 - t_m)z_w)$ and the labor hired in the market (n_d) from workers outside the family:

$$(2) \quad n = n_d + (1 - t_m)z_w,$$

where z_w is the working ability of the household. We denote as entrepreneurs the households that choose $t_m > 0$. Entrepreneurs, in turn, are partitioned into two subgroups depending on whether they hire outside labor or not. The first subgroup is given by the employers, who are those entrepreneurs hiring labor outside the family ($n_d > 0$). We assume that entrepreneurs that hire outside labor incur a fixed per period operating cost of c_f .³ The second subgroup are the self-employed, those entrepreneurs that only use their own household labor input ($n = (1 - t_m)z_w$ and $n_d = 0$). Workers are those households who use all their available time as workers ($t_m = 0$, obtaining labor earnings wz_w).

CAPITAL MARKETS

We assume that the financial intermediation industry is competitive. Intermediaries take deposits from households and pay the international interest rate r . They rent capital to entrepreneurs at a rate $r + \delta$ and loan employers the fixed cost of operation c_f . Enforcement problems limit the amount of borrowing and the capital rented to entrepreneurs. Following Buera, Kaboski and Shin (2011), entrepreneurs may renege on the contracts after production has taken place and keep a fraction $1 - \phi$ of the undepreciated capital and the revenue net of labor payments $(Y(m, k, n) - wn^d + (1 - \delta)k - c_f \mathbf{I}_{n_d > 0})$ but may lose the financial assets a deposited with the intermediary. Entrepreneurs that default regain access to the financial markets the following period. The parameter $\phi \in [0, 1]$ indexes the strength of the legal institutions in the economy, with $\phi = 1$ indicating perfect financial markets and $\phi = 0$ corresponding to an economy with no credit markets. We study equilibria in which financial contracts are restricted so that there is no default in equilibrium. This occurs when the amount of capital rented is limited by the largest amount $\bar{k}(a, z_m, z_w; \phi)$ consistent with entrepreneurs choosing to abide by their financial contracts. To characterize rental limits, consider the profit maximization problem of entrepreneurs that take as given the capital

³The fixed cost is introduced so that employers demand a non-trivial amount of labor (an amount bounded away from zero), thereby making the distinction between self-employed and employer meaningful.

k used in the business operation:

$$\begin{aligned} \pi(z_m, z_w, a; k) &\equiv \max_{n, n_d, t_m} \{m^\gamma k^\eta n^\theta - wn_d - r(k - a) + a - \delta k - c_f I_{n_d > 0}\} \\ &\text{subject to} \\ m &= t_m z_m, \\ n &= (1 - t_m) * z_w + n^d, \\ &\text{where } t_m \in [0, 1], n^d \geq 0, k \text{ given.} \end{aligned}$$

The following proposition extends results in Buera, Kaboski and Shin (2011) to characterize the rental limits $\bar{k}(a, z_m, z_w; \phi)$.

Proposition 1 *Capital rental k by an entrepreneur with wealth a and skills (z_m, z_w) is enforceable if and only if*

$$\begin{aligned} \pi(z_m, z_w, a; k) &\geq (1 - \phi) \max_{m, n, n_d, t_m} \{m^\gamma k^\eta n^\theta - wn_d + (1 - \delta)k - c_f I_{n_d > 0}\} \\ &\text{subject to} \\ m &= t_m z_m, \\ n &= (1 - t_m) * z_w + n^d, \\ &\text{where } t_m \in [0, 1], n^d \geq 0. \end{aligned}$$

The upper bound on capital rental that is consistent with entrepreneurs choosing to abide by their contracts can be represented by a function $\bar{k}(a, z_m, z_w; \phi)$, which is increasing in a , z_m , z_w and ϕ .

Proof. See Appendix A.

The income of an entrepreneur in state (z_m, z_w, a) making optimal production decisions, given prices and borrowing limits, is given by

$$\begin{aligned} y_e(z_m, z_w, a) &\equiv \max_k \{\pi(z_m, z_w, a; k)\} - a \\ &\text{subject to} \\ k &\leq \bar{k}(a, z_m, z_w; \phi) \end{aligned}$$

The income of a household that chooses to work for a wage is $y_w(z_m, z_w, a) = wz_w + ra$. Household income is the maximum between the entrepreneurial and workers' income:

$$y(z_m, z_w, a) = \max\{y_e(z_m, z_w, a), y_w(z_m, z_w, a)\}.$$

Households maximize expected discounted lifetime utility

$$\begin{aligned} & \max_{c_j, a_{j+1}} E \left\{ \sum_{j=1}^J \beta^j U(c_j) \right\} \\ & \text{subject to} \\ & c_j + a_{j+1} = y(z_{mj}, z_{wj}, a_j), \\ & c_j, a_{j+1} \geq 0, \end{aligned}$$

II. Time Allocation and Occupational Maps

We now study, in partial equilibrium (e.g. for a fixed wage rate), how our theory can give rise to three active occupational choices: worker, self-employed, and employer. We show that when capital markets are perfect, occupational choices are entirely determined by the ability ratio $\frac{z_w}{z_m}$. Individuals with a high $\frac{z_w}{z_m}$ ratio have a comparative advantage at working and choose to become workers, individuals with a low $\frac{z_w}{z_m}$ ratio have a comparative advantage at entrepreneurship and choose to become employers, and those with intermediate skill ratios prefer to be self-employed. We also characterize how tight borrowing constraints distort occupational choices and the rates of return to skills.

We start by analyzing the determinants of self-employment income. Self-employed individuals choose how much time to allocate to managerial versus working activities and how much capital to use in production. In Appendix A we show that the optimal time devoted to management by self-employed entrepreneurs (entrepreneurs not hiring outside labor) is $t_m^* = \frac{\gamma}{\gamma+\theta}$. The marginal product of their time is equated across its two uses (managerial and working time) and satisfies:

$$(3) \quad MPT_{se} = r_{mw}(\mu) (z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}, \quad \text{where } r_{mw}(\mu) = \left[\frac{\gamma^\gamma \eta^\eta \theta^\theta}{(r + \delta + \mu)^\eta} \right]^{\frac{1}{1-\eta}}$$

is the rate of return to the composite skill input $(z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}$ and μ is the Lagrange multiplier associated with the borrowing constraint. The marginal product of the self-employment time is the product of the skill composite $(z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}$ and the rate of return r_{mw} . The skill composite $(z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}$ is a geometric average of the managerial and working abilities of the self-employed individual. The rate of return $(r_{mw}(\mu))$ to the skill composite (z_{mw}) depends on the parameters of the production technology, the real interest rate (r) ,

and the Lagrange multiplier (μ) associated with the borrowing constraint. Note that the return to the skill composite decreases with (μ). Hence, borrowing constraints generate heterogeneity in the rates of return to skills among self-employed individuals. The income of a self-employed individual with assets a is given by

$$y_{se} = r_{mw}(\mu) (z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}} + \mu k + ra,$$

where $k = \bar{k}(z_m, z_w, a)$ and μ varies across the self-employed with different characteristics (z_m, z_w, a).

We now analyze the decisions of employers. Employers choose how much of their time to allocate to managerial versus working activities and how much capital (k) and (outside) labor services (n_d) to use in production. In Appendix A we show that there is a threshold level of asset holdings $a^*(z_m, z_w)$ such that for assets below this level the marginal product of entrepreneurial time is equal to wz_w and the time allocation problem of the employer features an interior solution in which the employer performs both managing and working activities. If asset holdings are higher than the threshold $a^*(z_m, z_w)$, then the marginal product of entrepreneurial time is higher than that as a worker and the time allocation problem exhibits a corner solution $t_m = 1$.⁴

The marginal product of employers' time (MPT_e) can be expressed as the product of managerial skills z_m and the rate of return $r_m(\mu)$ on the employer's managerial skill:

$$MPT_e = z_m r_m(\mu) \geq z_w w \text{ with strict inequality if } t_m = 1, \text{ where,}$$

$$(4) \quad r_m(\mu) = \gamma \left[\left(\frac{\eta}{(r + \delta + \mu)} \right)^\eta \left(\frac{\theta}{w} \right)^\theta \right]^{\frac{1}{1-(\eta+\theta)}}$$

Note that $r_m(\mu)$ is the rate of return to the managerial input z_m and μ is the Lagrange multiplier associated to the borrowing constraint. The income of an employer with ability (z_m, z_w) and assets a is given by

$$y_e = z_m r_m(\mu) + \mu k + ra - c_f,$$

where $k = \bar{k}(z_m, z_w, a)$. The rate of return ($r_m(\mu)$) depends on the parameters of the production technology, the real interest rate (r), and the Lagrange

⁴In our calibrated model economy, about 99% of employers choose $t_m = 1$.

multiplier (μ) associated with the borrowing constraint. Note that borrowing constraints (μ) generate heterogeneity in rates of return to skills among employers. We are now ready to characterize occupational choice decisions in an economy with perfect enforcement ($\phi = 1$).

Proposition 2: Perfect Enforcement. *Assume that $\phi = 1$. Let $R_1^* \equiv \left(\frac{r_{mw}^*}{w}\right)^{\frac{\theta+\gamma}{\gamma}}$ and $R_2^* \equiv \left(\frac{r_m^*}{r_{mw}^*}\right)^{\frac{\theta+\gamma}{\theta}}$, where r_{mw}^* and r_m^* are the rates of return to the skill composite $(z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}$ and the managerial skill defined in (3) and (4) when $\mu = 0$.*

- 1) *The rates of return on skills do not vary across individuals ($r_m = r_m^*, r_{mw} = r_{mw}^*$) so that income inequality is all due to heterogeneity in skills and assets.*
- 2) *If there is no fixed cost of operation of being an employer ($c_f = 0$), then the optimal occupational choice is the one that maximizes the marginal product of time and is only determined by the skill ratio $(\frac{z_w}{z_m})$ as follows: Individuals with an ability ratio $\frac{z_w}{z_m} > R_1^*$ work for a wage, individuals with $R_1^* > \frac{z_w}{z_m} > R_2^*$ are self-employed, and those with $\frac{z_w}{z_m} < R_2^*$ are employers.*
- 3) *If employers face a positive fixed cost of operation ($c_f > 0$), the decision to be an employer depends on the skill ratio $(\frac{z_w}{z_m})$ and on the absolute level of managerial ability (z_m). Individuals prefer to become employers relative to self-employment when the ability ratio is such that $\frac{z_w}{z_m} < R_2^* \left(1 - \frac{c_f}{z_m r_{mw}^*}\right)$.*

When there is perfect enforcement (and the fixed cost of operation is zero), occupational choices can be characterized in terms of two skill ratios R_1^* and R_2^* . Individuals with comparative advantage at working (an ability ratio $\frac{z_w}{z_m} > R_1^*$) work for a wage, individuals with $R_1^* > \frac{z_w}{z_m} > R_2^*$ are self-employed, and those with comparative advantage at managing ($\frac{z_w}{z_m} < R_2^*$) are employers.⁵ A positive fixed cost of operation ($c_f > 0$), implies that employers require a minimum scale in order to operate a profitable business so that the decision to be an employer depends both on the skill ratio $\frac{z_w}{z_m}$ and on the level of managerial ability z_m . Proposition 3 characterizes occupational choice decisions when borrowing constraints bind ($\phi < 1$).

Proposition 3: Imperfect Capital Markets *Assume that $\phi < 1$ and consider individuals for which the borrowing constraint binds ($\mu > 0$).*

⁵If parameter values are such that $R_1^* < R_2^*$, then the self-employment region is empty.

- 1) *The rates of return on skills (r_m, r_{mw}) vary across individuals and decrease with μ . Skill returns positively covary with asset holdings. Income inequality is due to heterogeneity in skills, assets, and rates of return.*
- 2) *Occupational choices depend on the skill ratio, assets, and the absolute level of skills (the last two affect the rates of return to skills because they determine the tightness of the borrowing constraint).*

Capital market imperfections distort returns to skills and, thus, occupational choices. A tight borrowing constraint depresses the rate of return to the managerial ability of employers and the return to the composite skill input supplied by self-employed individuals. It also increases the rate of return to capital faced by entrepreneurs. For fixed skills (z_m, z_w), an increase in assets makes the borrowing constraint less tight (μ), increasing the rates of return to skills and depressing the shadow cost of capital. As a result, asset holdings matter importantly for occupational choice decisions in the presence of financial frictions. Panel a in Figure 1 shows how occupational choices vary with skills in our calibrated model economy for an individual with assets equal to the median in the economy. Panel b draws the occupational map when perfect enforcement is introduced in the calibrated model economy. A comparison of the occupational maps, reveals that capital market imperfections expand the region where self-employment is optimal at the expense of the regions where employer and worker are the preferred occupational choices. Note that capital markets imperfections affect the sources of income inequality by causing heterogeneity in the rates of return and by making these returns covary with asset holdings.

III. Quantitative Analysis

A. Calibration

We partition the parameters in the model economy into two. The first group includes the parameters that are set using estimates from other studies in the literature. The second group consists of all the parameters that are calibrated by simulating the model economy.

Parameters set exogenously. The model period is set to a year. The international interest rate is set at 3%. The utility function is assumed to be of the CES type: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 1.5$. The parameters of the production function are set to standard values in the literature: $\gamma = .198, \eta = .3256, \theta = .4764$ (see Guner, Ventura and Xu (2008), Buera, Kaboski and Shin (2011)). The annual depreciation rate is set to $\delta = 0.06$.

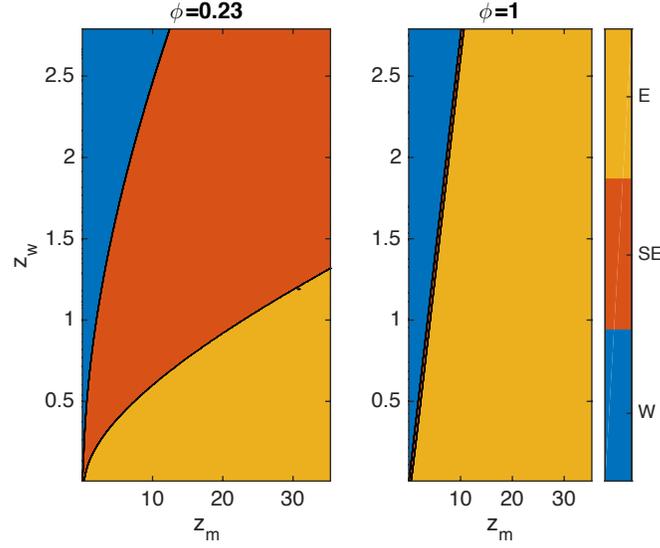


FIGURE 1. OCCUPATIONAL MAP

Calibrated parameters. For ease of exposition, below we list the parameters to be calibrated together with a corresponding target that helps identify each parameter. Nonetheless, it is important to keep in mind that the calibration is a multidimensional mapping in which all parameters and calibration targets are interrelated.

- 1) The discount factor β is chosen so that the capital to income ratio in the steady-state equilibrium is equal to 2.4, which is consistent with the capital to income ratio in Brazil (see Júnior et al. (2004)).
- 2) Enforcement of credit contracts ϕ to match a credit to GDP ratio of 43% in Brazil.⁶
- 3) The coefficients on the quartic polynomial on age, determining how the working and managerial skills vary with age, are set so that the age-profile of mean earnings for workers and entrepreneurs are roughly consistent with the data.
- 4) Following Storesletten, Telmer and Yaron (2005), the parameters determining the stochastic process on working ability such as the variance of fixed effects $\sigma_{\alpha_w}^2$, persistence of autoregressive process ρ_w , and

⁶We use the average Private Credit/GDP from 2003 until to 2010 from the World Development Indicators from the World Bank

the variance of the innovation to working ability over the life-cycle $\sigma_{\epsilon_w}^2$ to match the age profile of the variance of log wages.⁷

- 5) There are various parameters determining the stochastic process on managerial ability (i) the variance of fixed effects on managerial skills $\sigma_{\alpha_m}^2$; (ii) variance of innovations to managerial abilities ($\sigma_{\epsilon_m}^2$); and (iii) the persistence of the auto-regressive process on managerial ability (ρ_m). To pin down these parameters, we target: (i) the proportion of entrepreneurs and workers in the population of households (32% versus 68%); (ii) the variance of entrepreneurial log-earnings (1.06); and (iii) the persistence of being an employer over two consecutive years (70%). The more persistent the employer occupation is the easier it is for employers to accumulate retained earnings to finance their businesses, an effect that diminishes the quantitative impact of financial frictions diminishes. Hence, it is important that the calibrated model economy is consistent with the persistence of employers in the Brazilian data.
- 6) The parameter $\rho_{w,m}$ driving the correlation between managerial and working skills is pinned down by targeting the ratio of median earnings between entrepreneurs and workers.
- 7) The fixed cost of operation of employers c_f is set to match the fraction of employers among entrepreneurs (one fourth).

B. Calibration results

We now discuss how the calibrated model economy matches the calibration targets. The values of the calibrated parameters are reported in Table 1.

Table 2 shows that the model economy matches reasonably well the targets for the credit to GDP ratio of 43% and the capital to income ratio of 2.4. Figure 2 compares the variance of log-earnings of workers in the model economy with the Brazilian data. The model economy is consistent with the fact that there is a large amount of inequality early in the life cycle

⁷We consider a discrete approximation of the bivariate autoregressive process on skills that allows for a hundred pairs of skills (z_m, z_w) . Because of our life-cycle environment, the variance of shocks grow with age. The Markov chain is allowed to vary with age so that the finite state approximation of the autoregressive bivariate process matches the unconditional variance of the continuous bivariate shock process at each age. Regarding fixed effects, the bivariate normal distribution is discretized with 3 values for working skill and 5 values for managerial skills. As a result, there are 15 pairs of fixed effects. At each age, there are 1500 possible pairs of skills (z_w, z_m) . The numerical algorithm to compute the model economy is based on Fella (2014).

TABLE 1—CALIBRATED PARAMETERS

| $\rho_{w,m}$ | ρ_w | ρ_m | $\sigma_{\alpha,w}^2$ | $\sigma_{\alpha,m}^2$ | $\sigma_{\epsilon_w}^2$ | $\sigma_{\epsilon_m}^2$ | c_f | ϕ | β |
|--------------|----------|----------|-----------------------|-----------------------|-------------------------|-------------------------|-------|--------|---------|
| 0.1 | 0.98 | 0.78 | 0.38 | 1.59 | 0.03 | 0.99 | 0.10 | 0.23 | 0.995 |

and that inequality grows substantially with age over the life cycle. The stochastic process on working skills is characterized by a high persistence (ρ_w close to 1), which is needed to match the linear age-profile of the variance of log wages in the Brazilian data. This is consistent with the findings of Storesletten, Telmer, and Yaron (2005) for the US. Relatively to previous findings for the US economy, the calibration requires a large variance of individuals' fixed effects (σ_{α_w}) to match the high inequality of wages at age 20 in Brazil.

TABLE 2—CALIBRATION RESULTS-MODEL AGGREGATES

| | Data | Model |
|-----------------------------|------|-------|
| K/Y | 2.4 | 2.3 |
| Credit/GDP | 43% | 42% |
| Var Log(Earn)-Entrepreneurs | 1.1 | 1.2 |

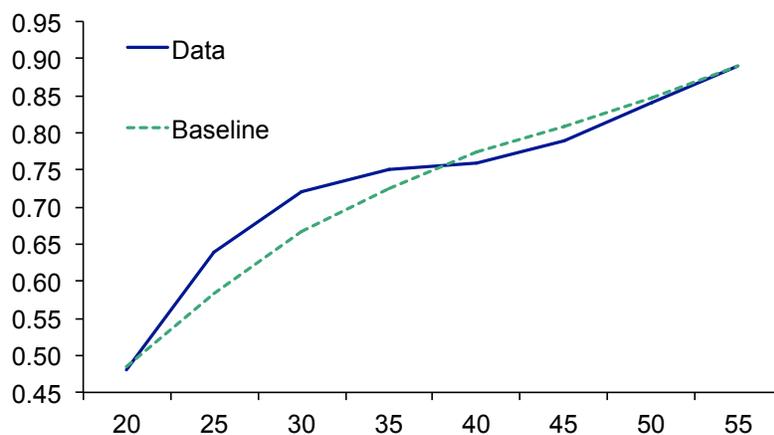


FIGURE 2. VARIANCE OF LOG(EARNINGS)-MODEL VS DATA

Source: Author's Elaboration based on Pesquisa Mensal de Emprego 2003-2010.

The calibration implies that the variances of fixed effects and of the innovation of managerial skills are much larger than the corresponding variances of working skills (see Table 1). This is necessary for the model economy to be consistent with the large variance of entrepreneurial earnings in the Brazilian data.

Table 3 compares the fraction of households that are workers, self-employed, and employers in the calibrated model economy and in Brazil. The model economy matches quite closely the fractions of workers (68%), self-employed (24%), and employers (8%) in the data. The calibration implies that employers faced a fixed cost of operation that is equivalent to 17% of the mean income of employers (43% of the mean income of all entrepreneurs). The calibration also matches the fact that about 70% of the employers in Brazil at a given point in time are still employers one year later (see Table 3). In the calibrated model economy the persistence of entrepreneurial shocks is high ($\rho_m = 0.78$), but less than the persistence of shocks on working ability ($\rho_w = 0.98$).

TABLE 3—CALIBRATION RESULTS—OCCUPATIONAL STRUCTURE

| Fraction | Data | Model |
|---------------|------|-------|
| Workers | 68% | 67% |
| Self-Employed | 24% | 24% |
| Employers | 8% | 9% |
| Emp to Emp | 70% | 68% |

The calibration implies that the correlation between skills is positive but moderate ($\rho_{w,m} = 0.1$). This is necessary for the economy to be consistent with the fact that the ratio of median income between entrepreneurs and workers is equal to 1 in the Brazilian data.⁸

Summing up, we believe that the calibrated model economy provides a reasonable account of income inequality and occupational choices in Brazil.

⁸In a sensitivity analysis, available upon request, we calibrate two economies in which we exogenously set a high and low skill correlation. We find that a high skill correlation implies a counterfactually high ratio of median income between entrepreneurs and workers. Conversely, a calibration with a negative skill correlation implies that the median income of entrepreneurs is below that of workers, which is also counterfactual.

C. Performance of the baseline economy.

We now discuss how the baseline economy matches some facts on occupational transitions and on the distribution of income across occupations that were not directly targeted in the calibration.

Table 4 reports predictions of the model economies on occupational transitions between the three occupations considered (worker, self-employed, employer). While there are 9 possible occupational transitions, we remind the reader that the calibration only targeted the persistence of being an employer over two consecutive years. The baseline economy matches the patterns on the persistence of occupational choices remarkably well (see Table 4). First, consistently with the data, the model economy predicts that being a worker is quite persistent: 89% of workers in the model economy are workers one year later. This percentage is about 94% in the data. Both in the baseline economy and in the data, entrepreneurs are less likely to remain in their occupation than workers. Second, examining transition rates within the entrepreneurial class, the calibrated model economy matches the fact that individuals are much more likely to transit into employer from self-employment than from being a paid worker. In the data, individuals are eight times more likely to become employers if they are self-employed rather than working for a wage. Indeed, the (annual) transition rate from self-employment into employer is 8% while the transition rate from worker to employer is only 1%. In the model, these rates are 9% and 1%, respectively. Third, the (annual) transition rates out of employer in the data imply that employers are much more likely to switch to self-employment (22%) than to paid work (9%). In the model economy, these transition rates are 24% and 7%, respectively.

The model economy was calibrated to match two statistics on the distribution of income across workers and entrepreneurs (ratio of median income and the variance of income of each of the two occupations). Figure 3 plots the distribution of income across these two occupation categories in the data and in the model.⁹ The model was calibrated to match the fact that the ratio of median income between entrepreneurs and workers is equal to one and that the variance of entrepreneurial income is higher than that of workers. Figure 4 plots the distribution of income across three occupations (e.g. the entrepreneurial category is subdivided in two groups: employers and self-employed). The baseline economy is consistent with the fact that there

⁹The plots show estimated Epanechnikov kernel densities to the Brazilian and model-simulated data.

TABLE 4—PERFORMANCE OF THE MODEL-TRANSITIONS

| Transitions | Data | Model |
|-------------|------|-------|
| E to W | 9% | 7% |
| E to SE | 22% | 24% |
| E to E | 68% | 70% |
| SE to W | 15% | 26% |
| SE to SE | 77% | 65% |
| SE to E | 8% | 9% |
| W to W | 94% | 89% |
| W to SE | 5% | 10% |
| W to E | 1% | 1% |

Note: In the table W stands for Workers, SE for Self-Employed and E for Employers.

is substantial income heterogeneity in all three occupations. Moreover, as in the data, the distribution of self-employment income is shifted to the left, relative to that of workers, and the distribution of worker’s income is shifted to the left, relative to that of employers. Nonetheless, we emphasize that there is substantial heterogeneity among employers: Some low income employers make less income than the median self-employed individual.

All in all, the model economy matches reasonably well patterns on occupational transitions and distribution of income within and across occupations. While we could have built a theory with three distinct skills (one for each occupation) to better match the facts, our theory with heterogeneity in two skills is consistent with the patterns in the data on the transition rates into and out of the employer occupation.

D. Experiment: Removing financial frictions

We now assess how financial frictions impact on occupational choices, resource allocation, aggregate output, and the distribution of income in the economy. This is done by comparing the equilibrium allocation of the baseline economy with that of an economy that is identical to the baseline economy but has perfect credit-enforcement institutions ($\phi = 1$).



FIGURE 3. DISTRIBUTION OF EARNINGS-DATA VS MODEL I

OCCUPATIONAL STRUCTURE AND FINANCIAL FRICTIONS

We find that removing financial frictions has important consequences on the occupational structure in the economy. Entrepreneurs are now selected in terms of their skills rather than their wealth, leading to an increase in the average productivity of employers. Moreover, capital is now reallocated towards more productive entrepreneurs who were the most likely to be constrained in the baseline economy. The better selection of entrepreneurs and the improved allocation of capital result in an increase in labor demand by the most productive employers and a higher equilibrium wage rate. This general equilibrium effect strongly discourages self-employment, as can be seen by comparing the occupational maps for the economy with $\phi = 1$ and the baseline economy (see Figure 1). The elimination of capital market imperfections reduces the region where self-employment is optimal in favor of the regions where employer and worker are the preferred occupational choices. Overall, we find that the fraction of entrepreneurs decreases from 33% to 18% and that the decrease in entrepreneurship is mostly due to a large decrease in the rate of self-employment (from 24% to 11%). The fraction of employers only drops by 2 percentage points. Hence, the theory accounts for the fact that changes in rates of entrepreneurship across rich

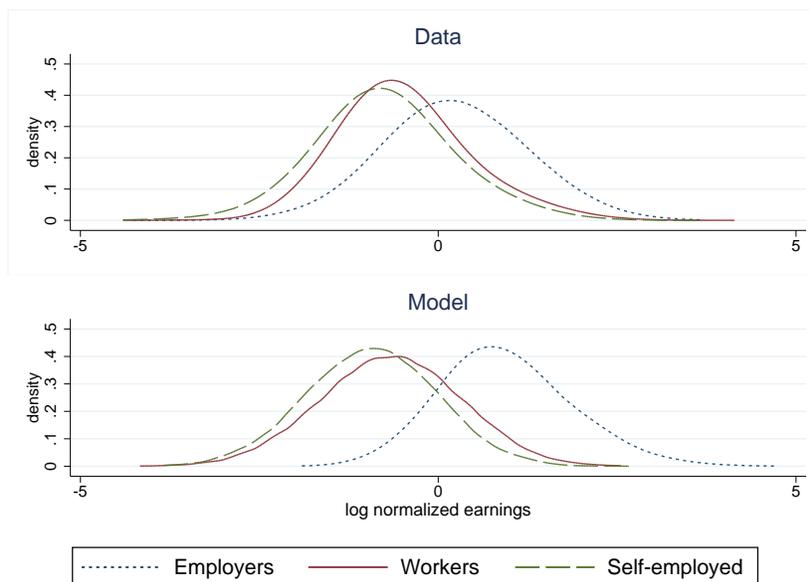


FIGURE 4. DISTRIBUTION OF EARNINGS-DATA VS MODEL II

and poor countries are mostly due to changes in the self-employment rate. The theory is also consistent with the fact that the fraction of workers in the labor force tends to increase with economic development: It increases from 68% in the baseline economy to 82% with perfect capital markets.

OUTPUT, RESOURCE ALLOCATION, AND FINANCIAL FRICTIONS

The elimination of financial frictions changes dramatically the contribution to total output from employers and self-employed entrepreneurs. While the aggregate production of employers increases by 64%, the output of the self-employed decreases by 53%. The large increase in the production of employers is due to the substantial increase in their average productivity and in the resources (capital and labor inputs) used by them. The fall in the output of self-employed individuals is mostly explained by the notable reduction in the number of self-employed entrepreneurs. The gain in aggregate output of eliminating financial frictions is about 48%, which is substantially smaller than the 64% increase in the aggregate output of employers. Below, we show that the consequences of financial frictions on the efficiency of production varies substantially between employers and self-employed entrepreneurs.

The aggregate production function of employers is:¹⁰

$$(5) \quad Y_E = \frac{\left[\sum_{i \in E} z_i r_i^{-\frac{\eta}{\gamma}} \right]^{1-\theta}}{\underbrace{\left[\sum_{i \in E} r_i^{\frac{\theta-1}{\gamma}} z_i \right]^\eta}_{TFP_E}} K_E^\eta L_E^\theta,$$

where r_i denotes the marginal product of capital (shadow cost of funds) of employer i , E indicates the set of employers, and (Y_E, K_E, L_E) stand, respectively, for the employers' aggregate output, capital input, and labor input. The term TFP_E denotes the employers' aggregate total factor productivity.

The aggregate production function (5) implies that the production efficiency (TFP_E) of employers is affected by: (a) the heterogeneity in the marginal product of capital (r_i) across employers; (b) the distribution of employers' managerial skills (z_i); and (c) the number of employers (mass of E). Financial frictions distort the allocation of capital across employers by introducing variation in the marginal product of capital across employers. We refer to the misallocation of capital as a distortion on the intensive margin of entrepreneurship. When capital is reallocated optimally across employers, so that $r_i = r$, the aggregate TFP of employers becomes:

$$(6) \quad TFP_E^R = \left(\sum_{i \in E} z_i \right)^{1-\theta-\eta} = \left(\sum_{i \in E} z_i \right)^\gamma,$$

where TFP_E^R indicates the level of production efficiency when capital is optimally reallocated across existing employers. TFP_E^R is determined by the sum of the managerial input across all employers. Financial frictions affect the level of TFP_E^R by distorting both the selection (average productivity) and the quantity of employers. We refer to the misallocation of the entrepreneurial input (managerial skill) as a distortion on the extensive margin of entrepreneurship.

The gain in production efficiency of removing financial frictions can be measured as the the log-difference between TFP in the economy with perfect enforcement ($TFP_{\phi=1}$) and in the baseline economy (TFP_B). The efficiency gains can take place along the intensive or extensive margin of entrepreneurship. The efficiency gains along the intensive margin accrue from correcting

¹⁰As shown in Appendix B, this aggregate production function can be derived by integrating the decision rules of employers.

the capital misallocation in the baseline economy (so that the marginal product of capital is equated across employers). The efficiency gains along the extensive margin accrue from correcting the misallocation of entrepreneurial talent in the baseline economy (misallocation of the managerial input in production). In Appendix B we show that the gains in production efficiency can be expressed as follows:

$$\begin{aligned}
 \log(TFP_{gains}) &= \log(TFP_{\phi=1}) - \log(TFP_B) \\
 &= \underbrace{\gamma (\log(\bar{z}_{\phi=1}) - \log(\bar{z}_B))}_{\text{avg TFP employers}} + \underbrace{\gamma (\log(N_{\phi=1}) - \log(N_B))}_{\text{number of employers}} \\
 &\quad \underbrace{\hspace{10em}}_{\text{ExtensiveMargin}} \\
 (7) \quad &+ \underbrace{\log \left(\sum_{i \in E_B} z_i \right)^\gamma - \log \frac{\left[\sum_{i \in E_B} z_i r_i^{-\frac{\eta}{\gamma}} \right]^{1-\theta}}{\left[\sum_{i \in E_B} r_i^{\frac{\theta-1}{\gamma}} z_i \right]^\eta}}_{\text{IntensiveMargin}}
 \end{aligned}$$

Panel A of Table 5 reports the aggregate production efficiency gain of eliminating financial frictions in the baseline economy, as well as the decomposition of the efficiency gains between the intensive and extensive margins of entrepreneurship. Note that the aggregate efficiency gain is about 9.0% and that the intensive margin (reallocation of capital) accounts for almost all of the gain (a gain of about 8.9%). Surprisingly, the extensive margin (misallocation of entrepreneurial talent) only accounts for a modest TFP gain of 0.1%.

In understanding the low efficiency gains along the extensive margin, it is useful to note that, as shown in (7), the TFP gains along the extensive margin result from the sum of two effects: (i) the change in the average managerial input of employers; (ii) the change in the number of employers. The elimination of financial frictions leads to an increase in the average quality (managerial ability) of employers of about 6.1%, which increases aggregate TFP. However, there is also a reduction in the number of employers of 6 percentage points, which decreases aggregate TFP of employers (see Table 5, Panel A). Intuitively, the decreasing returns to scale of the production technology in capital and labor introduces a love for variety in the aggregate production function. As a result, *aggregate* TFP depends on the aggregate

TABLE 5—ELIMINATING FINANCIAL FRICTIONS: EFFICIENCY GAINS IN PRODUCTION

| Panel A: Baseline Economy | | | | | |
|---------------------------|-------|-----------|-----------|----------|---------|
| TFP gains | Total | Intensive | Extensive | Avg. TFP | Nº Ent. |
| Employers | 9.0% | 8.9% | 0.1% | 6.1% | -6.0% |
| Self-employed | -4.5% | 6.3% | -10.9% | 4.3% | -15.2% |

| Panel B: Economy High-Fixed Costs | | | | | |
|-----------------------------------|-------|-----------|-----------|----------|---------|
| TFP gains | Total | Intensive | Extensive | Avg. TFP | Nº Ent. |
| Employers | 27.3% | 13.0% | 14.3% | 0.8% | 13.5% |
| Self-employed | 1.1% | 12.8% | -11.7% | 8.0% | -19.7% |

managerial input used in production, as shown in the aggregate production function in equation (5). Since the increase in the average skills of employers is compensated by a reduction in the mass of employers, the improved allocation of entrepreneurial talent does not increase the aggregate managerial input and, hence, TFP.

The impact of financial frictions on the TFP of self-employed entrepreneurs (TFP_{SE}) is quite different from that of employers. Indeed, we find that the elimination of financial frictions leads to a *decrease* in TFP_{SE} of 4.5%. Two striking patterns are important for this result: First, the gains from capital reallocation among the self-employed account for a TFP gain of 6.3%, which is smaller than the 8.9% efficiency gain attained by reallocating capital among employers. Second, the decrease in the number of self-employed individuals of 33% accounts for a *decrease* in TFP_{SE} of about 15.2% (see Appendix B for the decomposition of the TFP gains among the SE). The misallocation of capital is less important among the SE than employers because the average managerial input used by the SE is about one twentieth

that of employers. As a result, SE individuals tend to be entrepreneurs that operate their businesses at a low scale and are thus much less likely to be borrowing constrained than employers (about 85% of the SE are not credit constrained). On the other hand, the majority of employers face a binding borrowing constraint so that the marginal product of capital varies among them.

DISCUSSION

Our baseline economy implicitly features a technology adoption decision. When entrepreneurs switch from self-employment to employers, they increase their managerial time from 0.29 to 1. As a result, TFP increases by 28% after an occupational switch. Yet, not all entrepreneurs find it optimal to become employers: entrepreneurs with little net worth or with low managerial ability operate at a low scale and may not be able to cover the fixed cost of operation (c_e) faced by employers. We now show that the calibration of this parameter matters importantly for the impact of financial frictions on TFP in our model economy.

Our calibration procedure pins down the magnitude of c_e by targeting the fraction of employers in the Brazilian population. The calibrated model economy implies that expenditures in fixed costs are about 4.9% of GDP. To evaluate the sensitivity of our results to the value of fixed costs, we re-calibrate the model economy assuming that employers face a high fixed costs of operation. In particular, we keep all the targets of the baseline calibration but for the fraction of employers in the data. Instead, we target an expenditure share in fixed costs of 6.5% of GDP.

How much do the TFP gains of eliminating financial frictions vary across the calibrated model economies? We find that the TFP gains vary from 9.0% in the baseline economy to 27.3% in the economy with high fixed costs (compare the results in Panel A with Panel B in Table 5). The large difference in TFP gains across experiments is mostly due to the variation in TFP gains along the extensive margin. While the TFP gains on the extensive margin are 0.1% in the baseline economy, they are 14.3% in the economy with high fixed costs.

In understanding this result, it is important to note that the elimination of financial frictions *increases* the number of employers in the economy with high fixed costs whereas it *decreases* the number of employers in the baseline economy. In the economy with high fixed costs, the number of employers more than doubles (from 0.9% to 1.9% of the population) when financial

frictions are eliminated.¹¹ As a result, both the number and the average productivity of employers increase, which leads to a large increase of TFP along the extensive margin. On the contrary, in the baseline economy the number of employers decreases with the elimination of financial frictions (from 9.2% to 6.8% of the population). The reduction of the number of employers offsets the TFP gain due to the better selection of employers, explaining the low TFP gain along the extensive margin in the baseline economy.

We emphasize that self-employment plays an important role in diminishing the negative impact of financial frictions on aggregate output. This can be seen by shutting down self-employment in the baseline economy and simulating the removal of financial frictions. The output gain is now 53%, which is higher than the 48% increase obtained in the baseline economy. How can self-employment *decrease* the negative impact of financial frictions on aggregate output? Financial frictions make it hard for young and talented entrepreneurs (individuals with high managerial skills) to raise external funds. This effect is compounded by the fact that financial frictions depress the equilibrium real wage, which makes it difficult to accumulate savings by working for a wage. Talented entrepreneurs can circumvent the low wage by becoming self-employed and using their profits to build up savings, diminishing the negative impact of financial frictions on aggregate output. Importantly, the Brazilian data support that this mechanism might be important since 72% of the individuals that switch into the employer occupation over two consecutive years come *directly* from self-employment. The baseline economy matches this (non-targeted) moment well (predicting that 80% of flows into employment are from self-employment). Hence, both in the model and in the data SE represents an important pathway into becoming an employer.

¹¹This result resembles the findings of Midrigan and Xu (2014), who argue that the extensive margin is crucial for the efficiency gains of eliminating financial frictions. Midrigan and Xu (2014) consider an economy with both a modern technology and a traditional technology. The modern technology is assumed to be more productive than the traditional technology (they consider productivity improvements in the range of 20% to 40%). They assume that the adoption of the modern technology requires the payment of a large fixed cost. They target the aggregate amount of intangible investments in the Korean economy in order to pin down the costs of entry. Their calibration implies a fixed cost of adoption equivalent to 30 times annual profits in the traditional sector and a share of expenditures in fixed costs on GDP of 7.0%. The large adoption cost accounts for the fact that the elimination of financial frictions more than doubles the mass of modern producers, which is crucial for their finding that the extensive margin of entrepreneurship accounts for a large portion of the aggregate TFP gains.

FINANCIAL FRICTIONS AND THE DISTRIBUTION OF INCOME.

What are the quantitative effects of financial frictions on income inequality? We find that the Gini index of income decreases from 0.53 to 0.52 with the removal of financial frictions in our baseline economy ($\phi = 1$).¹² This modest variation in income inequality masks important differences across economies in the sources and persistence of income inequality.

Financial frictions have different effects on the distributions of the returns to capital and skills. We compute returns to skills following the derivations in Section II. In particular, returns to skills are computed as the total rents on working and managerial skills: wz_w for workers, $r_m z_m$ for employers, and $r_{mw} z_{mw}$ for self-employed. (See Section II for the definitions of r_m , r_{mw} , and z_{wm} .) We find that the Gini index of capital income is about 8 percentage points higher in the baseline economy than in the economy with perfect enforcement. On the other hand, the returns to skills are more evenly distributed in the baseline economy (Gini index of 0.52) than in the economy with no financial frictions (Gini index of 0.56). The opposite effects of financial frictions on the distributions of capital income and returns to skills offset each other and account for the small change in the Gini index of income.

Financial frictions reduce the inequality in skill returns by limiting the rents that talented individuals obtain from their skills. Since entrepreneurs with high managerial skills need to operate at a large scale to obtain all the potential rents from their skills, borrowing constraints diminish the rate of return attained by these individuals (see Proposition 3). On the other hand, talented individuals can obtain the full reward to their skills in the absence of financial frictions. Hence, the sources of income inequality differ across the baseline economy and the economy with perfect enforcement. Indeed, while the variation in skill returns explains 31% of the income variance in the baseline economy, it accounts for more than 81% of the income variance in the economy with no financial frictions (see Table 6). Capital income (directly) accounts for 24% of the income variance in the baseline economy, which is much higher than the 4.8% figure in the economy with $\phi = 1$. The covariance between capital and returns to skills plays a much more prominent role in the baseline economy than in the economy with no financial frictions, accounting for 44% of the income variance in the former economy

¹²It is interesting that the Gini index of (pre-tax and transfer) income in Brazil (0.52) is quite similar to the one in Germany (0.51) and France (0.50). The crucial difference across these countries is that tax and transfer programs have a large impact in reducing inequality in disposable income in the latter two countries but not in Brazil.

and a much lower 14% in the latter economy.

The effects of financial frictions on the distribution of income are particularly strong among employers. The variance of employers' log income in the baseline economy is about a half the one in the economy with perfect enforcement (0.47 versus 0.90). This fact is explained by the low variance of managerial returns in the baseline economy. The variance of managerial returns ($z_m r_m$) explains 25% of the variance of employers' income in the baseline economy, whereas it accounts for 90% of the employers' variation of income in the economy with $\phi = 1$ (see Table 6). On the other hand, capital income plays a much more important role in explaining the variation of employer's income in the baseline economy than in the economy with perfect enforcement. In the baseline economy capital income explains 27% of the employer's variance of income versus 2% in the perfect capital market economy. (see Table 6).

TABLE 6—VARIANCE DECOMPOSITION OF INCOME

| | All Households | | Employers | |
|-----------------|----------------|------------|--------------|------------|
| | Baseline Ec. | $\phi = 1$ | Baseline Ec. | $\phi = 1$ |
| Capital Income | 24% | 5% | 27% | 2% |
| Skill returns | 31% | 81% | 25% | 90% |
| Covariance Term | 44% | 14% | 48% | 8% |

The correlation between capital income and returns to skills is 0.80 in the baseline economy and decreases to 0.50 in the economy with perfect enforcement. When $\phi = 1$ capital income and returns to skills are positively correlated since high skill people tend to hold more capital than low ability people. The correlation between capital income and returns to skills is enhanced in the baseline economy, because financial frictions cause the rate of return to managerial ability to be positively correlated with asset holdings and, hence, with capital income. We also find that financial frictions increase the persistence of income. The persistence of income is measured by running the following regression in model simulated data

$$(8) \quad \log(y_{t,j}) = \alpha_j + \beta \log(y_{t-1,j}) + b_2 age_t + b_3 age_t^2,$$

where y_{tj} represents the income of individual j at age t , α_j is an individual fixed effect, and β measures the persistence of log-income. We find that removing financial frictions in the baseline economy reduces the estimated value of β from 0.81 to 0.74. Income is more persistent in the baseline economy because assets are positively correlated with rate of returns and because assets matter for occupational choices. On the other hand, in the economy with perfect enforcement the persistence of income is only driven by the persistence of shocks and asset holdings.

E. The political economy of financial frictions

Since our model economy was calibrated to Brazilian household data, we can use our theory to assess the distribution of the welfare gains of eliminating financial frictions in the Brazilian economy. We assume that the baseline economy is in a steady-state equilibrium and that suddenly and unexpectedly there is a once and for all institutional reform that increases ϕ to 1. On impact, the wealth distribution does not matter for occupational choice decisions. Now, the skill ratio $\frac{z_m}{z_w}$ is the only determinant of occupational choice decisions. Workers who could not operate as entrepreneurs because of binding borrowing constraints can now start a business. Moreover, entrepreneurs who were initially borrowing constraint see their managerial rents ($r_m z_m$) and entrepreneurial income increase (see the discussion in Section II). The importance of this effect varies across individuals: It is more important for talented entrepreneurs (high z_m) because, *ceteris paribus*, they were more likely to be initially constrained. As talented entrepreneurs raise their demand for capital relative to less talented entrepreneurs, capital is reallocated towards more productive entrepreneurs, increasing the demand for labor, and the equilibrium wage rate. The rise in wages decreases the profits of entrepreneurs. Hence, employers' income can go up or down depending on whether the increase in managerial rents is higher or lower than the increase in labor costs.

We compute the distribution of welfare gains for all individuals alive at the moment of the reform. For each household we compute the permanent consumption compensation in the original steady state that will let the household attain the same utility as in the perfect credit economy.¹³ We

¹³The small open economy assumption simplifies the computation significantly. Once the reform takes place, the marginal product of capital will be equated across entrepreneurs and will be equal to the international interest rate plus the depreciation rate of capital. On impact, competition for workers will drive the wage rate to its new long run value, which increases on impact by about 40%. While the distribution of wealth,

find that the average welfare gain among households alive at the period of the institutional reform is 16.5%. The standard deviation of the distribution of welfare gains is 13.5%. While the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Who are the households that lose with the reform?

We find that households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs at the period of the reform had the reform not taken place, and 66% would have been employers.

Employers are a positive selection from the population distribution of managerial skills. Then, the fact that about two thirds of those who oppose the reform are employers explains why the managerial ability of those supporting the reform is higher than that of those opposing the reform. Nonetheless, not all employers are against the reform: About 36% of employers in the initial equilibrium benefit from the elimination of enforcement problems. We find that the employers benefiting from the reform tend to be of a higher managerial ability than those who oppose it. The reason is that high-ability employers are more likely to be borrowing constrained than low-ability entrepreneurs. As a result, they are more likely to operate at an inefficient scale and to gain more from the elimination of enforcement problems. On the other hand, the financial reform hurts many of the lower-skill employers and forces them to change their occupation status: About 46% of the entrepreneurs that oppose the reform and would have been employers had the reform not taken place, do not hire any labor after the reform (most of them become self-employed after the reform). The wage hike after the reform makes it unprofitable for these entrepreneurs to hire outside labor.

In summary, while most households benefit from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform.

IV. Conclusions

We propose a parsimonious extension of the Lucas (1978) that distinguishes between entrepreneurs that are self-employed from those that are

consumption, and income may change for some periods after the reform, all macroeconomic aggregates (capital, GDP, wage rate) will be constant after the initial period of the reform.

employers (hire labor outside the household). The innovation of our theory is to model households that are heterogeneous in two skills – managerial and working skills – and to assume that entrepreneurs decide how much time to devote to the supply of labor and/or managerial input. The model is calibrated to household-level data from Brazil. The baseline economy is consistent with household data on the occupation structure, the distribution of income across occupations, and occupational transitions. Moreover, the theory accounts for the fact that the variation in rates of entrepreneurship between poor and rich countries is explained by the high rates of self-employment in poor countries, a fact that existing theories of occupational choice cannot account for.

We find that the elimination of financial frictions in Brazil would increase aggregate output by 48% and TFP by 9.0%. The increase in TFP is mostly due to the reallocation of capital across entrepreneurs (intensive margin of entrepreneurs) so that the extensive margin of entrepreneurship is not important for the effect of financial frictions on aggregate TFP in our baseline economy.

We show that the fixed cost of operation faced by employers is crucial for the quantitative impact of financial frictions on the extensive margin of entrepreneurship. When entrepreneurs switch from self-employment to employers, they increase their managerial time from 0.29 to 1, increasing their TFP by 28% upon an occupational switch. Yet, not all entrepreneurs find it optimal to become employers: entrepreneurs with little net worth or with low managerial ability operate at a low scale and may not be able to cover the fixed cost of operation faced by employers. Hence, the fixed cost of operation represents, implicitly, a technology adoption cost. Our calibration procedure pins down the magnitude of fixed costs by targeting the fraction of employers in the Brazilian population. In a counterfactual experiment in which we calibrate the model economy to a high fixed cost of operation, we find that the elimination of financial frictions lead to TFP gains along the extensive margin of 14.3%, which is much higher than the 0.1% gain in the baseline economy.

Interestingly, self-employment reduces the output losses caused by credit market imperfections in our baseline economy. Talented entrepreneurs can circumvent low wages by using self-employment as a pathway towards becoming an employer. The occupational transitions in the Brazilian household data support this prediction of the theory: the self-employed are nine times more likely to become employers than paid workers are and more than 70% of the transitions into employer status in two consecutive years come

directly from self-employment. Our baseline economy matches these facts closely.

We find that income from skills is more evenly distributed in the economy with imperfect capital markets than in the economy with no financial frictions. This is because financial frictions depress the rents earned by highly able entrepreneurs relative to an economy with perfect capital markets. Finally, we gain some insights into the political economy of capital market imperfections by studying the welfare gains and losses of reforming capital markets in our calibrated model economy. Our findings suggest that employers may have a vested interest in maintaining a status quo with low enforcement.

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APPENDIX A: PROOFS OF PROPOSITIONS .

Proof of Proposition 1. Capital rental k by an entrepreneur with wealth a and skills (z_m, z_w) is enforceable if and only if

$$\begin{aligned} \max_{m,n,n_d,t_m} \{m^\gamma k^\eta n^\theta - wn_d - r(k - a) + a - \delta k - c_f \mathbf{I}_{n_d > 0}\} \geq \\ \max_{m,n,n_d,t_m} \{m^\gamma k^\eta n^\theta - wn_d + (1 - \delta)k - c_f \mathbf{I}_{n_d > 0}\} \end{aligned}$$

which is equivalent to

$$(1 + r)a \geq \phi \left[\frac{1 - \phi + r + \delta + \delta\phi}{\phi} k - \max_{m,n,n_d,t_m} \{m^\gamma k^\eta n^\theta - wn_d - c_f \mathbf{I}_{n_d > 0}\} \right]$$

Following arguments in Buera, Kaboski and Shin (2011), the set of enforceable levels of capital rentals is characterized by a simple set of rental limits. Two cases are relevant. If the max in the RHS is attained with $n_d = 0$, the set of enforceable levels of capital is $[0, \bar{k}(a, z_m, z_w; \phi)]$ where $\bar{k}(a, z_m, z_w; \phi)$, where $\bar{k}(a, z_m, z_w; \phi)$ is given by unique root of the equation

$$(1+r)a = \phi \left[\frac{1 - \phi + r + \delta + \delta\phi}{\phi} k - \max_{m, n, n_d, t_m} \{m^\gamma k^\eta n^\theta - wn_d - c_f I_{n_d > 0}\} \right]$$

If the max in the RHS is attained with $n_d > 0$, then there are two positive roots of the above equation and the set of enforceable levels of capital rental is $[\underline{k}(a, z_m, z_w; \phi), \bar{k}(a, z_m, z_w; \phi)]$, where $\underline{k}(a, z_m, z_w; \phi)$ represents the smallest root. Nonetheless, the optimal production plan of the entrepreneur coincides with the solution to the individual problem subject to the simpler limit $k \leq \bar{k}(a, z_m, z_w; \phi)$. It can also be shown that $\bar{k}(a, z_m, z_w; \phi)$ is strictly increasing in a, z_m, ϕ and weakly (strictly) increasing in z_w (if $n_d = 0$). QED.

Before proceeding to the proofs of Proposition 2 and 3, it is convenient to characterize the production plan of self-employed and employers.

Self-employed: Production Plan. The optimal production plan of self-employed individuals maximize

$$\pi_{se} = (z_m t_m)^\gamma k^\eta (z_w (1 - t_m))^\theta - (r + \delta)k + (1 + r)a + \mu_k(\bar{k} - k)$$

where $\gamma + \eta + \theta = 1$. The FOC imply:

$$\{t_m\} \quad z_m^\gamma k^\eta z_w^\theta [\gamma t_m^{\gamma-1} (1 - t_m)^\theta - t_m^\gamma \theta (1 - t_m)^{\theta-1}] = 0 \Rightarrow t_m^* = \frac{\gamma}{\gamma + \theta}$$

$$\{k\} \quad (z_m t_m)^\gamma \eta k^{\eta-1} (z_w (1 - t_m))^\theta - r - \delta - \mu_k = 0 \Rightarrow k = \left[\frac{(z_m t_m)^\gamma \eta (z_w (1 - t_m))^\theta}{r + \delta + \mu_k} \right]^{\frac{1}{1-\eta}}$$

Note that the first FOC equates the marginal product of entrepreneurial time at managing and working. Combining the FOC we obtain that the marginal product of entrepreneurial time satisfies:

$$\begin{aligned} MPT_{se} &= \gamma z_m^\gamma (t_m^*)^{\gamma-1} k^{*\eta} (z_w (1 - t_m^*))^\theta \\ &= r_{mw} (z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}}, \\ (A1) \quad \text{where } r_{mw}(\mu) &= \gamma \eta^{\frac{\eta}{1-\eta}} \left(\frac{\gamma \theta}{(\gamma + \theta)^2} \right)^{\frac{\theta}{1-\eta}} \left(\frac{1}{r + \delta + \mu} \right)^{\frac{\eta}{1-\eta}}. \end{aligned}$$

Income of self-employed individuals can then be written as

$$\begin{aligned} y_{se} &= MPt_m t_m + MPt_w t_w + MPK k + ra - k(r + \delta), \\ y_{se} &= r_{mw}(\mu) (z_m^\gamma z_w^\theta)^{\frac{1}{\gamma+\theta}} + \mu k + ra. \end{aligned}$$

Employers: Production Plan. The optimal production plan of employers solves

$$\begin{aligned} \pi(z_m, z_w, a) &= \text{Max}_{t_m, t_w, n_d, k} (z_m t_m)^\gamma k^\eta (n_d + z_w t_w)^\theta - w n_d - (r + \delta)k + (1 + r)a \\ &k \leq \bar{k}, t_m + t_w = 1, t_w \geq 0. \end{aligned}$$

The non-negativity constraint on t_w ensures that managerial time cannot be bigger than 1. Associate the multiplier μ_k to the borrowing constraint, μ_t to the time constraint, and μ_{tw} to the non-negative constraint on the working time. The FOC of the problem imply

$$\begin{aligned} MPK &= (z_m t_m)^\gamma \eta k^{\eta-1} (n_d + z_w t_w)^\theta = r + \delta + \mu_k, \\ MPn_d &= (z_m t_m)^\gamma k^\eta \theta (n_d + z_w t_w)^{\theta-1} = w, \\ MPt_m &= z_m \gamma (z_m t_m)^{\gamma-1} k^\eta (n_d + z_w t_w)^\theta = \mu_t, \\ MPt_w &= (z_m t_m)^\gamma \theta k^\eta (n_d + z_w t_w)^{\theta-1} z_w = \mu_t - \mu_{tw}, \end{aligned}$$

where we have assumed that parameters are such that it is optimal to hire outside labor ($n_d > 0$). Combining the FOC we obtain:

$$w z_w = MPt_w \leq MPt_m, \text{ with equality only if } t_w > 0.$$

We divide the analysis into three steps.

Step 1: We first show that if the borrowing constraint does not bind ($\mu_k = 0$), then the entrepreneur allocates all his time to managerial tasks ($t_w = 0, t_m = 1$). Assume that $\mu_k = 0$ and let $n \equiv n_d + z_w(1 - t_m)$. Furthermore, to find a contradiction assume that $t_w > 0$. Then, $\mu_{tw} = 0$ implies $MPt_m = MPt_w$ so that

$$(A2) \quad z_m \gamma n = t_m z_m \theta z_w \Rightarrow t_m = \frac{\gamma n}{\theta z_w}.$$

Combining the FOC for MPK and MPn_d , gives

$$(A3) \quad (z_m t_m)^\gamma \left(\frac{w \eta n}{(r + \delta) \theta} \right)^\eta \theta n^{\theta-1} = w.$$

Combining (A2)-(A3) gives

$$(A4) \quad n^{\gamma+\theta+\eta-1} \left(\frac{z_m \gamma}{\theta z_w} \right)^\gamma \left(\frac{w \eta}{\theta(r+\delta)} \right)^\eta \theta = w,$$

which is false in general given that $\gamma + \theta + \eta - 1 = 0$. We conclude that if the borrowing constraint does not bind, then an employer optimally chooses to devote all his time to managerial tasks.

Step 2: Assume that the borrowing constraint binds ($k = \bar{k}$). We now show that there exists a threshold level of assets $a^*(z_m, z_w)$ such that the optimal production plan features $t_w > 0$ if $a < a^*(z_m, z_w)$ and $t_w = 0$ if $a > a^*(z_m, z_w)$. Thus, if the borrowing constraint is not too tight, employers allocate all their time to managerial activities. We now find conditions for which $t_m < 1$ (or, equivalently, $t_w > 0$). Note that $t_m < 1$ only if $\mu_{tw} = 0$. In this case, the marginal product of entrepreneurial time is equated across the two uses of time. From the FOC it can be obtained that

$$MPt_w = MPt_m \Rightarrow n = \frac{\theta z_w t_m}{\gamma}.$$

Plugging n into the FOC with respect to labor demand and solving for t_m gives an expression for the optimal fraction of time dedicated to managerial tasks:

$$t_m = \left[\frac{\theta z_m^\gamma \bar{k}^\eta}{w} \left(\frac{\gamma}{z_w \theta} \right)^{1-\theta} \right]^{\frac{1}{1-\gamma-\theta}}.$$

Note that $t_m < 1$ iff

$$\bar{k}(a, z_m, z_w) < k^*(z_m, z_w) \equiv \left[\frac{w}{\theta z_m^\gamma} \left(\frac{z_w \theta}{\gamma} \right)^{1-\theta} \right]^{\frac{1}{\eta}}.$$

Since $\bar{k}(a, z_m, z_w)$ is increasing in a , the inverse of this function can be used to define a threshold level of assets $a^*(z_m, z_w)$ such that $t_m < 1$ if and only if assets are below this threshold. Otherwise, $t_m = 1$.

Step 3: Compute the marginal product of employers' time. From Step 1 and 2, when assets are below $a^*(z_m, z_w)$ we have $MPt_m = MPt_w = w z_w$. On the other hand, when assets are above $a^*(z_m, z_w)$, $t_m = 1$ and $MPt_m > MPt_w$. To obtain an expression for MPt_m note that the FOC with respect

to capital and outside labor imply:

$$k = \frac{w\eta}{(r + \delta + \mu_k)\theta} n_d$$

$$n_d = \left(\frac{\theta z_m^\gamma}{w} \left[\frac{w\eta}{(r + \delta + \mu_k)\theta} \right]^\eta \right)^{\frac{1}{1-(\eta+\theta)}}$$

Plugging k and n_d into $MPt_m = \gamma z_m^\gamma k^\eta n_d^\theta$ gives

$$MPt_m = z_m \gamma \left[\left(\frac{\eta}{(r + \delta + \mu)} \right)^\eta \left(\frac{\theta}{w} \right)^\theta \right]^{\frac{1}{1-(\eta+\theta)}} = z_m r_m(\mu)$$

(A5) where $r_m(\mu) = \gamma \left[\left(\frac{\eta}{(r + \delta + \mu)} \right)^\eta \left(\frac{\theta}{w} \right)^\theta \right]^{\frac{1}{1-(\eta+\theta)}}$.

We now study occupational choice decisions. We start by considering the decision between working for a wage vs. becoming self-employed.

Worker vs. Self-employment. An individual with ability (z_m, z_w) prefers to be self-employed rather than work for a wage if and only if

$$z_w w + ra < (z_m^\gamma z_w^\theta)^{\frac{1}{\theta+\gamma}} r_{mw}(\mu) + \mu k + ra,$$

which holds when the skill ratio satisfies

$$(A6) \quad \frac{z_w}{z_m} < \left[\frac{r_{mw}(\mu) + \mu k / (z_m^\gamma z_w^\theta)^{\frac{1}{\theta+\gamma}}}{w} \right]^{\frac{\theta+\gamma}{\gamma}}.$$

Self-employment vs. Employer. An individual with ability (z_m, z_w) and assets a prefers being an employer rather than self-employment if and only if

$$(z_m^\gamma z_w^\theta)^{\frac{1}{\theta+\gamma}} r_{mw}(\mu_{se}) + \mu_{se} k_{se} + ra < z_m r_m(\mu_e) + \mu_e k_e + ra,$$

where μ_e and μ_{se} are the Lagrange multipliers associated with the borrowing constraints when the individual is an employer or is self-employed, respectively, and k_e and k_{se} are the capital used in production in these occupations.

This inequality holds when the ability ratio is such that

$$(A7) \quad \frac{z_w}{z_m} < \left[\frac{r_m(\mu_e)}{r_{mw}(\mu_{se})} + \frac{(\mu_e k_e - c_f)}{z_m r_{mw}(\mu_{se})} - \frac{(\mu_{se} k_{se})}{z_m r_{mw}(\mu_{se})} \right]^{\frac{\theta+\gamma}{\theta}}.$$

Proof of Proposition 2 (Economy with perfect enforcement).

When $\phi = 1$ the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$).

Part 1. Setting $\mu = 0$ in (A1) and (A5), it follows that the rates of return to skills (r_{mw} and r_m) do not vary across individuals. Denote these returns by r_{mw}^* and r_m^* . Since individuals face the same skill prices, income inequality is all due to heterogeneity in skills and asset holdings. This proves part 1 of the proposition.

Part 2. Setting $\mu = 0$ in (A6) implies that the individual prefers to be self-employed rather than work for a wage if and only if

$$\frac{z_w}{z_m} < \left(\frac{r_{mw}^*}{w} \right)^{\frac{\theta+\gamma}{\gamma}} \equiv R_1^*.$$

Setting $c_f = 0$ and $\mu = 0$ in (A7) implies that the individual prefers to be an employer rather than be self-employed if and only if

$$\frac{z_w}{z_m} < \left(\frac{r_m^*}{r_{mw}^*} \right)^{\frac{\theta+\gamma}{\theta}} \equiv R_2^*.$$

This establishes the result in part 2.

Part 3. Now consider $c_f > 0$. Setting $\mu = 0$ in (A7) implies that the individual prefers to be an employer rather than be self-employed if and only if

$$\frac{z_w}{z_m} < R_2^* \left(1 - \frac{c_f}{z_m^* r_{mw}^*} \right)^{\frac{\theta+\gamma}{\theta}}.$$

The decision between being a worker or self-employed is not affected by the fixed cost of operation c_f . This establishes the result in part 3.

Proof of Proposition 3 (Economy with imperfect enforcement).

Consider an individual with a binding borrowing constraint (otherwise, occupational choice decisions and returns to skills are characterized as in Proposition 2). From (A1) and (A5), it follows that rates of return to skills (r_{wm} and r_m) decrease with the tightness of the borrowing constraint (e.g. returns decrease with μ). Since rates of return to skills vary across

individuals, income inequality is due to heterogeneity in skills, assets, and rates of return. From (A6) and (A7), it follows that occupational choices now depend on the skill ratio, asset holdings, and the absolute level of skills (the last two matter because they affect the value of μ).

APPENDIX B: ALGEBRA ON TFP.

Let $m_i \equiv (z_m t_m)$ be the managerial input supplied by entrepreneur i . His/her output i is given by:

$$(B1) \quad y_i = m_i^\gamma k_i^\eta n_i^\theta.$$

Denote by r_i and w_i the marginal product of capital and labor of entrepreneur i :

$$(B2) \quad r_i = \eta m_i^\gamma k_i^{\eta-1} n_i^\theta$$

$$(B3) \quad w_i = \theta m_i^\gamma k_i^\eta n_i^{\theta-1}$$

Dividing (B3) by (B2) gives

$$(B4) \quad \frac{k_i \theta}{n_i \eta} = \frac{w_i}{r_i} \Rightarrow n_i = \frac{\theta r_i}{\eta w_i} k_i.$$

Plugging (B4) into (B2) gives:

$$(B5) \quad k_i = \left[\frac{\eta}{r_i} \left(\frac{\theta r_i}{\eta w_i} \right)^\theta \right]^{\frac{1}{\gamma}} m_i$$

Combining (B4) with (B5) we get

$$(B6) \quad n_i = \frac{\theta r_i}{\eta w_i} \left[\frac{\eta}{r_i} \left(\frac{\theta r_i}{\eta w_i} \right)^\theta \right]^{\frac{1}{\gamma}} m_i$$

From (B5)

$$(B7) \quad \frac{k_i}{K} = \frac{k_i}{\sum_i k_i} = \frac{r_i^{\frac{\theta-1}{\gamma}} w_i^{-\frac{\theta}{\gamma}} m_i}{\sum_i r_i^{\frac{\theta-1}{\gamma}} w_i^{-\frac{\theta}{\gamma}} z_i}$$

$$(B8) \quad \frac{n_i}{N} = \frac{n_i}{\sum_i n_i} = \frac{r_i^{-\frac{\eta}{\gamma}} w_i^{\frac{\eta-1}{\gamma}} m_i}{\sum_i r_i^{-\frac{\eta}{\gamma}} w_i^{\frac{\eta-1}{\gamma}} z_i}$$

Combining (B1), (B7), and (B8), output of entrepreneur i can be written as:

$$(B9) \quad y_i = z_i^\gamma \frac{\left[r_i^{\frac{\theta-1}{\gamma}} w_i^{-\frac{\theta}{\gamma}} z_i \right]^\eta \left[r_i^{-\frac{\eta}{\gamma}} w_i^{\frac{\eta-1}{\gamma}} z_i \right]^\theta}{\left[\sum_i r_i^{\frac{\theta-1}{\gamma}} w_i^{-\frac{\theta}{\gamma}} m_i \right]^\eta \left[\sum_i r_i^{-\frac{\eta}{\gamma}} w_i^{\frac{\eta-1}{\gamma}} m_i \right]^\theta} K^\eta N^\theta$$

Aggregate output can be obtained by adding output across entrepreneurs (ENT):

$$(B10) \quad Y = \sum_{i \in ENT} y_i = \frac{\sum_{i \in ENT} m_i^\gamma \left[r_i^{\frac{\theta-1}{\gamma}} w_i^{-\frac{\theta}{\gamma}} m_i \right]^\eta \left[r_i^{-\frac{\eta}{\gamma}} w_i^{\frac{\eta-1}{\gamma}} m_i \right]^\theta}{\underbrace{\left[\sum_{i \in ENT} r_i^{\frac{\theta-1}{\gamma}} w_i^{-\frac{\theta}{\gamma}} m_i \right]^\eta \left[\sum_{i \in ENT} r_i^{-\frac{\eta}{\gamma}} w_i^{\frac{\eta-1}{\gamma}} m_i \right]^\theta}_{TFP_{ENT}}} K^\eta N^\theta$$

B1. Employers

We now focus on the subset of entrepreneurs that are employers. Employers equate the marginal product of labor to the market wage $w_i = w$. From (B10), the aggregate TFP of employers (adding over $i \in E$) is:

$$(B11) \quad TFP_E = \frac{\left[\sum_{i \in E} m_i r_i^{-\frac{\eta}{\gamma}} \right]^{1-\theta}}{\left[\sum_{i \in E} r_i^{\frac{\theta-1}{\gamma}} m_i \right]^\eta}$$

When capital is reallocated optimally across employers, so that $r_i = r$, the aggregate TFP of employers becomes:

$$(B12) \quad TFP_E^R = \left(\sum_{i \in E} m_i \right)^{1-\theta-\eta} = \left(\sum_{i \in E} m_i \right)^\gamma$$

Denoting by $TFP_{\phi=1}$ and TFP_B the total factor productivity under perfect enforcement ($\phi = 1$) and in the baseline economy, the efficiency gains of removing financial frictions can be expressed as:

$$\begin{aligned} \log(TFP_{gains}) &= \log(TFP_{\phi=1}) - \log(TFP_B) \\ &= \log \left(\sum_{i \in E_{\phi=1}} m_i \right)^\gamma - \log \frac{\left[\sum_{i \in E_B} m_i r_i^{-\frac{\eta}{\gamma}} \right]^{1-\theta}}{\left[\sum_{i \in E_B} r_i^{\frac{\theta-1}{\gamma}} m_i \right]^\eta} \\ &= \log \left(\sum_{i \in E_{\phi=1}} m_i \right)^\gamma - \log \left(\sum_{i \in E_B} m_i \right)^\gamma + \log \left(\sum_{i \in E_B} m_i \right)^\gamma - \log \frac{\left[\sum_{i \in E_B} m_i r_i^{-\frac{\eta}{\gamma}} \right]^{1-\theta}}{\left[\sum_{i \in E_B} r_i^{\frac{\theta-1}{\gamma}} m_i \right]^\eta} \\ &= \underbrace{\gamma (\log(\bar{m}_{\phi=1}) - \log(\bar{m}_B))}_{\text{avg TFP employers}} + \underbrace{\gamma (\log(N_{\phi=1}) - \log(N_B))}_{\text{number of employers}} \dots \\ &\quad \dots + \underbrace{\log \left(\sum_{i \in E_B} m_i \right)^\gamma - \log \frac{\left[\sum_{i \in E_B} m_i r_i^{-\frac{\eta}{\gamma}} \right]^{1-\theta}}{\left[\sum_{i \in E_B} r_i^{\frac{\theta-1}{\gamma}} m_i \right]^\eta}}_{K \text{ misallocation}} \end{aligned}$$

The above formula shows that the gain in aggregate TFP of removing financial frictions is driven by three terms: First, the change in the average productivity of employers, the change in the number of employers, and the gains from removing capital misallocation due to financial frictions.

B2. Self-employed

Output by self-employed i is given by

$$(B13) \quad y_{SE}^i = (z_m^i t_m^i)^\gamma (k^i)^\eta [z_w^i (1 - t_w^i)]^\theta$$

The marginal product of capital is then

$$(B14) \quad r^i = (z_m^i t_m^i)^\gamma \eta (k^i)^{\eta-1} [z_w^i (1 - t_w^i)]^\theta$$

so that solving for capital gives

$$(B15) \quad k^i = \left\{ \frac{(z_m^i t_m^i)^\gamma \eta [z_w^i (1 - t_w^i)]^\theta}{r_i} \right\}^{\frac{1}{1-\eta}}$$

Using that all self-employed make the same choice for t_m^i we obtain that the share of self-employed aggregate capital used by i is:

$$(B16) \quad s_k^i = \frac{k^i}{\sum_i k^i} = \frac{k^i}{K_{SE}} = \frac{\left\{ \frac{1}{r_i} (z_m^i)^\gamma (z_w^i)^\theta \right\}^{\frac{1}{1-\eta}}}{\sum_i \left\{ \frac{1}{r_i} (z_m^i)^\gamma (z_w^i)^\theta \right\}^{\frac{1}{1-\eta}}} = \frac{\left(\frac{1}{r_i} \right)^{\frac{1}{1-\eta}} z_{mw}^i}{\sum_i \left(\frac{1}{r_i} \right)^{\frac{1}{1-\eta}} z_{mw}^i}$$

where $z_{mw} = \left\{ (z_m^i)^\gamma (z_w^i)^\theta \right\}^{\frac{1}{1-\eta}}$ denotes the composite self-employment input.

The share of labor input by self-employed i is:

$$(B17) \quad s_n^i = \frac{z_w^i (1 - t_m^i)}{N_{SE}} = \frac{z_w^i (1 - t_m^i)}{\sum_i z_w^i (1 - t_m^i)} = \frac{z_w^i}{\sum_i z_w^i}.$$

Combining (B13), (B16), and (B17) the output of self-employed individual

i can be expressed as:

$$\begin{aligned}
y_{SE}^i &= (z_m^i t_m^i)^\gamma (k^i)^\eta [z_w^i (1 - t_w^i)]^\theta \\
&= (z_m^i t_m^i)^\gamma \left[\frac{(\frac{1}{r_i})^{\frac{1}{1-\eta}} z_{mw}^i}{\sum_i (\frac{1}{r_i})^{\frac{1}{1-\eta}} z_{mw}^i} \right]^\eta K_{SE}^\eta \left[\frac{z_w^i}{\sum_i z_w^i} \right]^\theta N_{SE}^\theta \\
&= (z_m^i t_m^i)^\gamma \left[\frac{(\frac{1}{r_i})^{\frac{1}{1-\eta}} (z_m^i)^{\frac{\gamma}{1-\eta}} (z_w^i)^{\frac{\theta}{1-\eta}}}{\sum_i (\frac{1}{r_i})^{\frac{1}{1-\eta}} z_{mw}^i} \right]^\eta K_{SE}^\eta \left[\frac{z_w^i}{\sum_i z_w^i} \right]^\theta N_{SE}^\theta \\
&= (t_m^i)^\gamma \frac{(\frac{1}{r_i})^{\frac{\eta}{1-\eta}} (z_m^i)^{\frac{\gamma}{1-\eta}} (z_w^i)^{\frac{\theta}{1-\eta}}}{\left[\sum_i (\frac{1}{r_i})^{\frac{1}{1-\eta}} z_{mw}^i \right]^\eta \left[\sum_i z_w^i \right]^\theta} K_{SE}^\eta N_{SE}^\theta \\
&= (t_m^i)^\gamma \frac{(\frac{1}{r_i})^{\frac{\eta}{1-\eta}} z_{mw}^i}{\left[\sum_i (\frac{1}{r_i})^{\frac{1}{1-\eta}} z_{mw}^i \right]^\eta \left[\sum_i z_w^i \right]^\theta} K_{SE}^\eta N_{SE}^\theta
\end{aligned}$$

Adding over all self-employed gives aggregate output by self-employed individuals (and using t_m constant across individuals SE):

$$\sum_{i \in SE} y_{SE}^i = \frac{(t_m)^\gamma \sum_{i \in SE} (\frac{1}{r_i})^{\frac{\eta}{1-\eta}} z_{mw}^i}{\underbrace{\left[\sum_{i \in SE} (\frac{1}{r_i})^{\frac{1}{1-\eta}} z_{mw}^i \right]^\eta \left[\sum_{i \in SE} z_w^i \right]^\theta}_{TFP_{SE}}} K_{SE}^\eta N_{SE}^\theta$$

Aggregate TFP of SE under capital reallocation becomes

$$TFP_{SE}^R = \frac{(t_m)^\gamma \left[\sum_{i \in SE} z_{mw}^i \right]^{1-\eta}}{\left[\sum_{i \in SE} z_w^i \right]^\theta}$$

The TFP gains of removing financial frictions in the self-employment sector

can be written as:

$$\begin{aligned}
\log(TFP_{gains}) &= \log \frac{\left[\sum_{i \in SE_{\phi=1}} z_{mw}^i \right]^{1-\eta}}{\left[\sum_{i \in SE_{\phi=1}} z_w^i \right]^\theta} - \log \frac{\sum_{i \in SE_B} \left(\frac{1}{r_i}\right)^{\frac{\eta}{1-\eta}} z_{mw}^i}{\left[\sum_{i \in SE_B} \left(\frac{1}{r_i}\right)^{\frac{1}{1-\eta}} z_{mw}^i \right]^\eta \left[\sum_{i \in SE_B} z_w^i \right]^\theta} \\
&= \log \frac{\left[\sum_{i \in SE_{\phi=1}} z_{mw}^i \right]^{1-\eta}}{\left[\sum_{i \in SE_{\phi=1}} z_w^i \right]^\theta} - \log \frac{\left[\sum_{i \in SE_B} z_{mw}^i \right]^{1-\eta}}{\left[\sum_{i \in SE_B} z_w^i \right]^\theta} \dots \\
&+ \log \frac{\left[\sum_{i \in SE_B} z_{mw}^i \right]^{1-\eta}}{\left[\sum_{i \in SE_B} z_w^i \right]^\theta} - \log \frac{\sum_{i \in SE_B} \left(\frac{1}{r_i}\right)^{\frac{\eta}{1-\eta}} z_{mw}^i}{\left[\sum_{i \in SE_B} \left(\frac{1}{r_i}\right)^{\frac{1}{1-\eta}} z_{mw}^i \right]^\eta \left[\sum_{i \in SE_B} z_w^i \right]^\theta} \\
&= \underbrace{(1-\eta) \left(\left[\log(\bar{z}_{mw}^{SE_{\phi=1}}) - \log(\bar{z}_{mw}^B) \right] - \theta \left[\log(\bar{z}_w^{\phi=1}) - \log(\bar{z}_w^B) \right] \right)}_{\text{quality entrepreneurial composite input}} \dots \\
&+ \underbrace{(1-\eta-\theta) \left[\log(\bar{SE}^{\phi=1}) - \log(\bar{SE}^B) \right]}_{\text{number of SE}} + \dots \\
&+ \underbrace{\log \frac{\left[\sum_{i \in SE_B} z_{mw}^i \right]^{1-\eta}}{\left[\sum_{i \in SE_B} z_w^i \right]^\theta} - \log \frac{\sum_{i \in SE_B} \left(\frac{1}{r_i}\right)^{\frac{\eta}{1-\eta}} z_{mw}^i}{\left[\sum_{i \in SE_B} \left(\frac{1}{r_i}\right)^{\frac{1}{1-\eta}} z_{mw}^i \right]^\eta \left[\sum_{i \in SE_B} z_w^i \right]^\theta}}_{\text{Kreallocation}}
\end{aligned}$$

APPENDIX C: DATA.

We use the Pesquisa Mensal de Emprego (PME) for the years 2003 until 2010. The PME is a monthly household survey covering the metropolitan areas of six Brazilian regions: Rio de Janeiro, São Paulo, Porto Alegre, Belo Horizonte, Recife and Salvador. Each individual is followed for three months, left out of the sample the next eight months and interviewed again the following 4 months. We take the first and fifth interview of each individual for the years 2003 until 2010. In this way we keep two observation of each individual, which corresponds to the same month of consecutive years. We keep only households where the head is male and he is older than twenty and younger than sixty years old. The earnings of the household are the sum of the earnings of all members. In order to make the earnings comparable we deflect them with the corresponding month Consumer Price Index (CPI) and we divide them by the number of adults equivalents in the house. In addition, we only keep individuals who are employed in both periods of

the survey. In the final data set we have 131,056 households with data for earnings. The individual household's age is defined as the age of the household head. We use 5 years bin, centered at the age of interested, in order to compute statistics by age. To do the transition matrix on occupations we consider the individual data. The variable of earnings that we consider is a constructed variable, which includes the earnings effectively perceived by the individual in the month from all the works done.